WHAT DO YOUNG CHILDREN KNOW ABOUT LINEAR MEASUREMENT AND IS IT RELATED TO CONSERVATION?

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For the past twenty years Piagetian theory has dominated mathematics and science curricula. In primary schools, teaching - learning sequences and activities in mathematics have traditionally been built around the ages and stages promulgated by Piaget as being appropriate to the particular performance level of cognition a child might have attained. Conservation - according to Piaget, is the precondition for all measurement. Consequently, exposure to 'real' measurement has usually been delayed until Years 2 or 3 in primary schools. Recent research indicates that children have quite well developed notions about measurement prior to their ability to conserve. This paper examines procedures that were developed to successfully test the ability of young children to measure length. Findings from a study carried out between 1985 and 1987 indicate that the ability to conserve is linked to the ability to measure, the knowledge of particular dimensional adjectives and experience with concrete material. The new NSW K - 6 Curriculum in Mathematics, released in 1989, reflects the findings of this research and has changed the theoretical basis of the teaching - learning sequences in the curriculum.

RESEARCH BASE FOR THE STUDY OF LINEAR MEASUREMENT

Research has indicated that children possess a broader measurement concept at an earlier age than that postulated by Piaget and continuously maintained by the neo-Piagetians.

- Philp,Kelly and Birch (Unpublished)
  Established a hierarchical order for the acquisition of dimensional terms (length, substance, weight). Three relevant conservation structures have a sine qua nonrelationship with the relevant scales of terms.

- Kelly, Philp and Lewis (1982)
  Investigated horizontal decalages, use of dimensional adjectives and a concept of measurement. They found that a concept of measurement appeared before children were able to conserve length.

- Carpenter (1971, 1975, 1976, 1981)
  Children appeared to be able to measure at a younger age than that predicted by Piaget. Measurement operations used appropriately may facilitate conservation judgements.

- Hiebert (1981, 1984)
  Sought to identify specific developmental abilities which are required to learn mathematical concepts - particularly measurement.

- Boulton - Lewis (1987)
  Examined the length measuring sequence established in literature in light of recent theories of cognition concerning information processing in children.

RELEVANCE OF THE STUDY TO AUSTRALIAN EDUCATION IN EARLY MATHEMATICS

Previous research has concentrated on what constitutes a valid concept of measurement and its distinction from measurement skills. This study seeks to establish whether the variables:

- concept of measurement
- conservation of length
- use of selected dimensional adjectives

are developmentally linked and whether the conceptions of teachers may influence the teaching of measurement in the lower primary grades. At present it appears that little research relating the results of clinical studies involving measurement materials and its place in the Mathematics syllabus exists in Australia.

MEASUREMENT AND CONSERVATION OF LENGTH STUDY

Sample

126 subjects in grades 1, 3 and 5 were selected from 5 schools in Sydney representing inner-city, suburban and outer-suburban areas. The age range was 5.11 - 11.4. The sample included children of mixed ethnicity and socio-cultural backgrounds although these variables were not specific delimiters of the study. The sample was structured to include children in the following categories:

1. Prior to the onset of conservation
2. Without conservation ability
3. With conservation ability

Subjects were selected at random by teachers.

Tests

All tests involved the use of concrete materials and objects easily manipulated by the children included in the study. Materials included cuisenaire rods stained black, pieces of dowel and ribbon and small tables of different dimensions. The six tests included:

- Two standard conservation of length tests
- Two concept of measurement tests
- Two dimensional adjectives tests

Results

Several types of statistical analyses were used in the study. Principally, a non-parametric analysis of variance based on a revised version of a program of multi-level chi-square analysis developed by Mitchell and Kelly of Macquarie University (Kelly, 1987) and a probabilistic model for the validation of hierarchies present in dichotomous data sets developed by Bailey (CONNECT:1988) of Sydney University.

Figures 1 and 2 examine the ability/inability to conserve length and measure length within the sample.

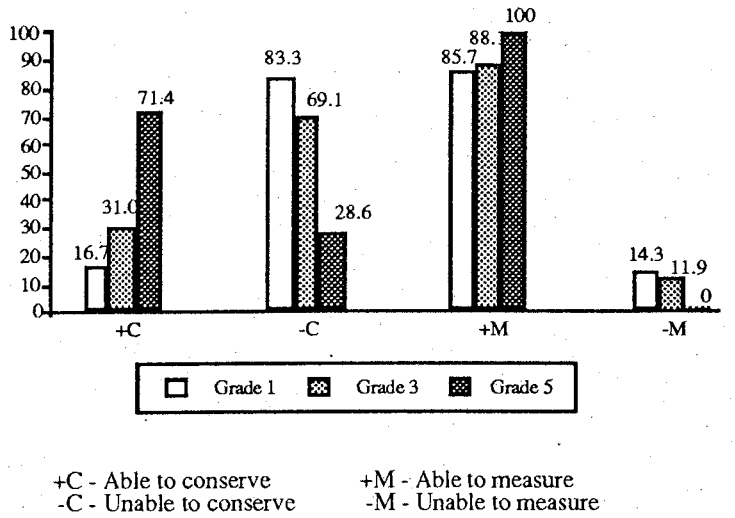


Figure 1: Conservation and measurement shown by grade.

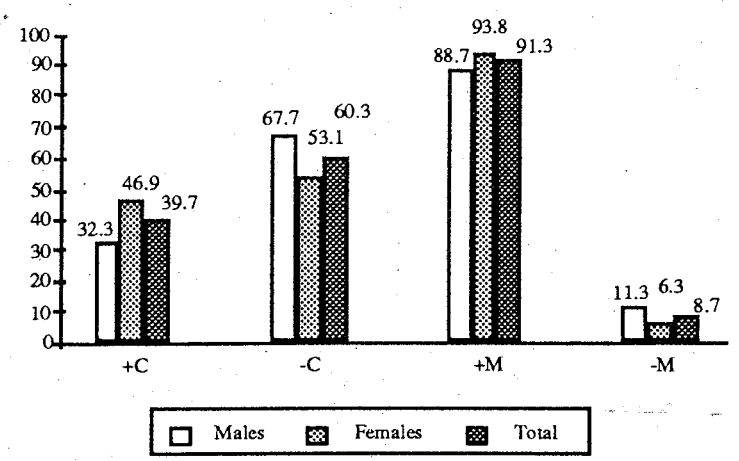


Figure 2: Conservation and measurement shown by sex.

Figure 1 shows that a greater proportion of subjects were able to measure than conserve. The results indicate that the ability to conserve is clearly related to age, as shown by the step-like progression in the +C column of Figure 1. This concurs with the Piagetian position. The ability to measure appears to be distinctly different. Figure 1 indicates a less pronounced step-like progression between grades.

Figure 2 examines the differences between sexes in the ability to conserve and measure. In keeping with research (Kelly et al., 1982; Boulton-Lewis, 1987), sex appears to have more of a significant effect on the ability to conserve than on the ability to measure.The four conditions which ensue as a result of the relationship between conservation and measurement were considered according to age and grade. The results are shown in Figure 3.

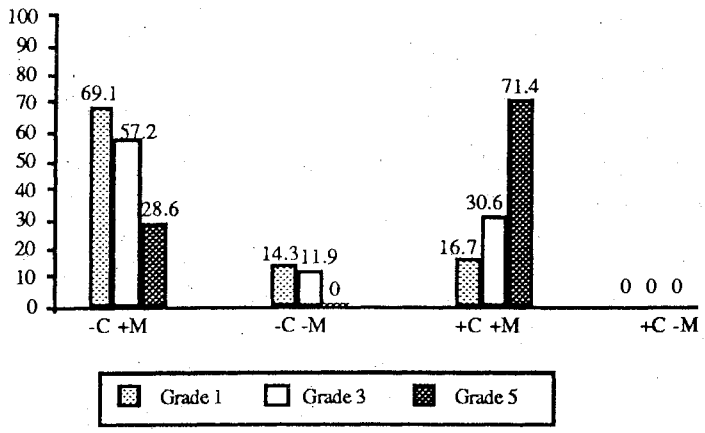


Figure 3: Four Conditions of Conservation and Measurement Shown by Grade

The -C +M column in Figure 3 represents those subjects who failed to conserve but were able to measure, the +C -M column represents those subjects who were able to conserve but failed to measure. Together the columns indicate that no subject was able to conserve without also succeeding on the measurement tasks.

In order to establish if the relationship between the variables was significant, the data was subjected to single and multi-level chi-square analysis. Table 1 summarises the results of the multi-level analysis.

Table 1: Summary of Significant Chi-Square Relations found in a Multi-Level analysis between the variables Conservation of Length and Measurement of Length.

Variables	χ^2	DF	P	C
First Order				
Grade x Conservation	36.55	4	$p < .001$	0.47
Grade x Measurement	8.27	4	$.05 < p < .01$	0.24
Conservation x Measurement	12.05	4	$.01 < p < .02$	0.30
Second Order				
Grade x Age x Conservation	36.55	8	$p < .001$	0.47
Third Order				
Grade x Age x Con. x Meas.	26.55	16	$.02 < p < .05$	0.41

Given the results obtained in the cell -C +M, and that the dependent variables conservation of length and measurement of length were reasonably significant at the first and third order levels of the chi-square analysis, the posited hypothesis that conservation of length is not possible without first being able to measure was accepted.

The Ability of Children to Measure

Figure 4 and Table 2 show that most subjects were able to measure. Significantly, a high proportion of age categories 1 and 2 containing the youngest subjects, were able to measure.

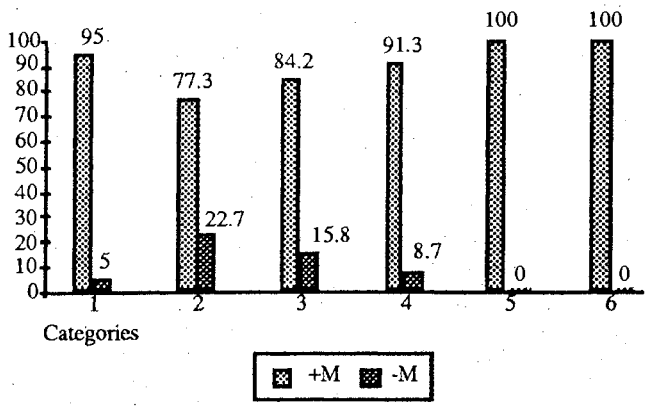


Figure 4: Measurement shown by age categorisation

Of critical importance in the consideration of measurement as a precondition for conservation of length is the ability of categories 1 and 2 to measure, since these 42 subjects are younger in age than that posited by Piaget as being capable of measuring meaningfully.

Table 2: Ability to Measure Shown by Age Categorisation and Test Type

Age Category	Sex	Rod Test Only	Table Test Only	Passed Both	Failed Both	Total
1	M	0	0	11	1	12
(5.11 - 6.6 years)	F	0	0	8	0	8
2	M	1	2	5	0	8
(6.7 - 7.2 years)	F	2	0	12	0	14
3	M	2	0	6	0	8
(7.11 - 8.5 years)	F	0	1	10	0	11
4	M	1	0	12	0	13
(8.6 - 8.11 years)	F	0	1	9	0	10
5	M	0	0	12	0	12
(9.11 - 10.8 years)	F	0	0	18	0	18
6	M	0	0	9	0	9
(10.9 - 11.4 years)	F	0	0	3	0	3
All Male		4	2	55	1	62
All Female		2	2	60	0	64
All Subjects		6	4	115	1	126

Table 2 describes in detail the ability of subjects to measure, including those subjects deemed transitional measurers, that is, those subjects who only passed one of the two measurement tests. One subject failed both tests and 10 subjects were transitional measurers. In category 1, 95% of subjects successfully measured, with 1 child failing both tests. In category 2, 77% of subjects successfully measured, 14% passed the Rod test only, 9% passed the Table test only and no-one failed both tests. The 42 subjects in categories 1 and 2 constitute the grade 1 sample aged between 5.11 and 7.2. 86% of grade 1 were able to measure, 12% were transitional measurers and 2% failed to measure.

The measurement tests required children to justify their length decisions with appropriate strategies supported by reasons for their choice. The tests were a strong, positive indication that subjects were able to measure using a strategy with perceptual or numerical clues and justify their actions with a reason. Figure 5 represents the reasons or strategies given by subjects to demonstrate the ways unit blocks would be useful to measure the lengths of the miniature tables (Measurement Test 2).

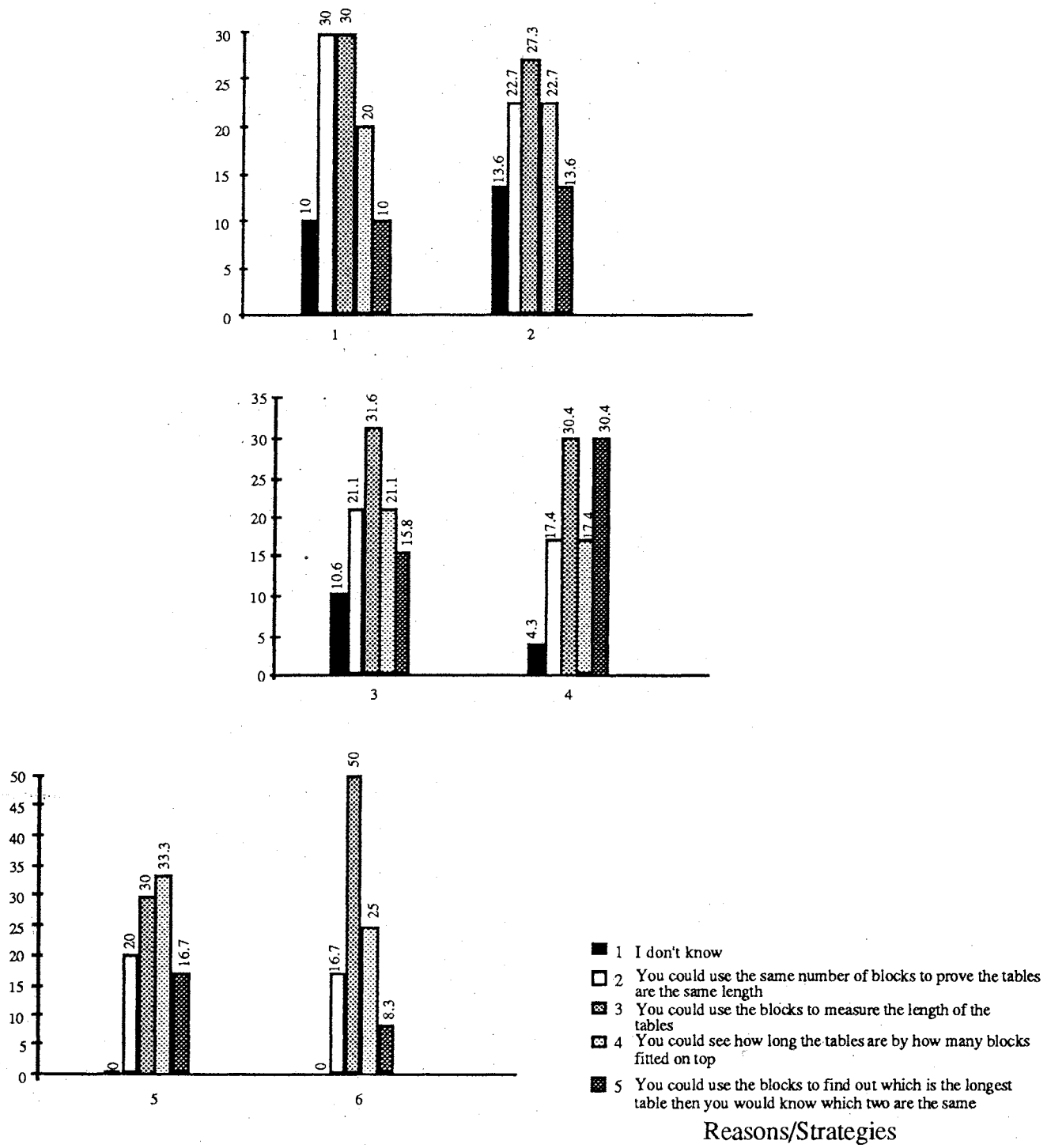
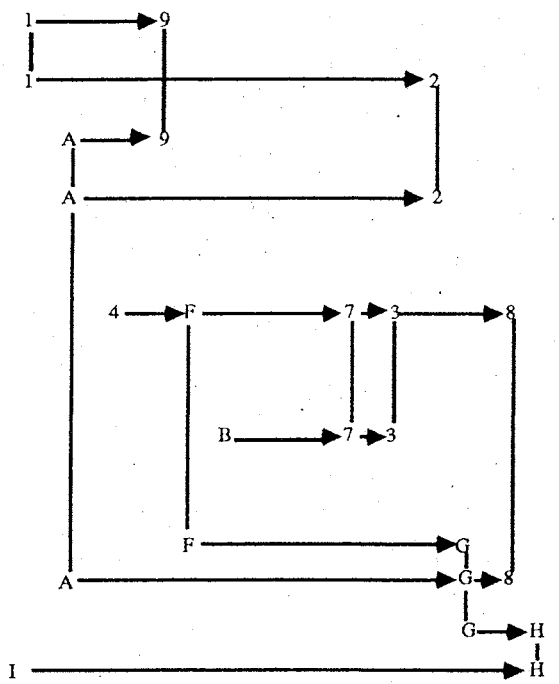


Figure 5: Reasons or Strategies Nominated by Subjects to Use Unit Blocks to Compare the Lengths of Two Tables Shown by Age Categorisation

Strategy 3, 'You could use the blocks to measure the length of the tables', gained the highest percentage of subjects in all age categories. Other strategies gained varied responses across the age categories. The measurement tests showed that subjects aged between 5.11 and 11.4 were able to measure using strategies with perceptual or numerical cues and justify their actions with a reason. The nature of the strategies given by subjects indicates that children younger than age 8 are able to measure meaningfully.

Conservation of Length, Measurement and the Use of Dimensional Adjectives

In order to establish the relationship between the selected dimensional adjectives, conservation of length and measurement, a program devised by Bailey of Sydney University based on the Dayton and Macready (1976) probabilistic model for the validation of hierarchies was used. A critical value for the sequencing statistic was established at 2.88, representing a p=.001 level of significance for 1 tailed tests. Figure 6 shows the sequence digraph for the probability matrix.



Variables

1 Big	6 As small as	B Longer than	G Not as short
2 Small	7 Not as big	C Shorter than	H Conservation
3 Bigger	8 Not as small	D As long as	I Measurement
4 Smaller	9 Long	E As short as	
5 As big as	A Short	F Not as long	

Figure 6: Sequence Digraph for the Probability Matrix of the Relationships between the Variables, Measurement, Selected Dimensional Adjectives and Conservation of Length.

The sequence graphically drawn in Figure 6 represents the linkages found to exist between the variables expressed in the three tasks. Figure 6 shows the sequence of difficulty between the easiest item *'big'*, to the hardest item, *conservation*. Two patterns are revealed in the data set.

1. Most of the selected dimensional adjectives precede conservation of length and are linked in sequence.

2. Measurement precedes and is linked to conservation of length but is not linked to the selected dimensional adjectives.

Table 3 shows the levels of significance found to exist between variables in the probabilistic matrix.

Table 3: . Levels of Significance for 1 Tailed Z Scores for Single Relations between variables in the Probabilistic Matrix.

Level of significance														
	.05	.03	.02	.01	.006	.005	.003	.002	.001	.0009	.0007	0005	.0002	.0001
Variable Pairs	A+B	9+5	2+H	6+H	F+E	1-A	D+H	C+B	F+7	A+G	F+G	3+8	G+H	1+9
	D+2	C+H		B+G	5+H	C+F	E+H	F+6	F+3			G+8	B+8	A+9
				F+B		7+G	4-B		7+3			4+7	1+2	D+5
				F+D		F+8			1+8				A+2	D+E
									I+H				B+7	D+6
													4+F	5+E
														5+6
														E+6
														4+3
														4+8
														7+8
Total Pairs	2	2	1	4	2	4	3	2	5	1	1	3	6	11

Variable List

1 Big	7 Not as big	D As long as
2 Small	8 Not as small	E As short as
3 Bigger	9 Long	F Not as long
4 Smaller	A Short	G Not as short
5 As big as	B Longer than	H Conservation
6 As small as	C Shorter than	I Measurement

The ability to measure was found to be a polar variable to the ability to conserve in terms of facility. The strength of the association found between the variables would strongly suggest that measurement be considered as a precondition for conservation. An identifiable order of hierarchical structure was found to exist between the ability to measure length and the ability to conserve length between the knowledge of select adjectives of extent and the ability to conserve length. Within the hierarchical structure both measurement and knowledge of select adjectives of extent preceded conservation of length.

REFERENCES

Bailey, M. (1978). *The analysis of sequential structures in educational data.* Unpublished PhD Thesis, Macquarie University.

Boulton-Lewis, G.M. (1987). Recent cognitive theories applied to sequential length measuring knowledge in young children. *British Journal of Educational Psychology,* 57, pp. 330-342.

Carpenter, T.P. (1971). *The performance of 1st and 2nd grade students on a non-standard set of measurement tasks.* Wisconsin Research and Development Centre for Cognitive Learning. Technical Report 211.

Carpenter, T. (1975). Measurement concepts of 1st and 2nd grade students. *Journal of Research in Mathematics Education,* 6, pp. 3-13.

Carpenter, T.P. (1976). Analysis and synthesis of existing research on measurement. In R. Lesh (Ed.) *Number and measurement.* USA: ERIC.

Carpenter, T.P. (1981). Research in cognitive development. In R.J. Shumway (Ed.) *Research in Mathematics Education.* Virginia: National Council of Teachers of Mathematics.

CONNECT, (1988). Fortran program developed by M Bailey of Sydney University for IBM PC Version of Probabilistic Hierarchy Model.

Dayton, M.C., & Macready, G.B. (1976). A probabilistic model for validation of behavioural hierarchies. *Psychometrika,* 41 (2), pp. 189-204.

Hiebert, J. (1981). Cognitive development and learning linear measurement. *Journal for Research in Mathematics Education,* 12, pp. 197-211.

Hiebert, J. (1984). Why do some children have trouble learning measurement concepts? *Arithmetic Teacher,* 31 (7), 19-24.

Kelly, M., (1987). Program Mitkel (plus plus: 5 variable chi-square analysis). Sydney: Macquarie University Computing Centre.

Kelly, M., Philp, H., & Lewis, J., (1982). Cognitive development and language: Some work in progress. *The Australian Journal of Education,* 26 (1), pp. 20-41.

Philp, H., Kelly, M., & Birch, R. (unpublished). A further exploration of the hierarchies of dimensional terms: The concept of measurement and conservation of length. Macquarie University.

Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15,* 105-127.

Willis, A. (1990). *The place of measurement operations in cognitive development.* Unpublished PhD Thesis, Macquarie University.

Willis, A. (1992). Linear measurement. In B. Atweh and J. Watson, (Eds.) *Research in mathematics education in Australasia 1988-1991.* Queensland University of Technology: Kelvin Grove Campus.